

***Intensified Algebra I* Program Design**

Intensified Algebra I aims to transform thinking about how to teach algebra to underprepared students. The program is designed for approximately 80 minutes of daily classroom instruction, and it presents a unique, coherent program that incorporates into algebra instruction ideas from social psychology and other areas that historically reside outside of the domain of typical algebra classes but are fundamentally important to students' success. Central to the design of the *Intensified Algebra I* program is the understanding that students who struggle in mathematics need more than simply more instructional time to be successful. Thus, in addition to a rigorous mathematics core, our program addresses the social, affective, linguistic, and strategic cognitive and metacognitive dimensions of learning mathematics.

Key design features

The key design features of *Intensified Algebra I* – derived from two years of planning and proof-of-concept development by the project's design team – are described below.

- **Rigorous algebra curriculum targeting students' conceptual understanding, associated skills, and related problem-solving and reasoning capabilities.** The core algebra instruction is built upon a technology-based delivery system developed by Agile Mind. The curriculum includes rigorous treatment of topics that are most critical for success in algebra and future mathematics courses, and targets students' conceptual understanding, associated skills, and related problem-solving and reasoning capabilities. *Intensified Algebra I* uses technology-based representations – visualizations, animations, and simulations – to allow students to explore algebra situations, increase their attention to mathematically important aspects of those situations, explicitly show relationships, and connect ideas.
- **Efficient and effective review/repair strategies.** Connecting new learning with prior knowledge involves both engaging students in tasks and activities that help them access relevant prior knowledge when they need it and “repairing” misconceptions they might have related to that knowledge. *Intensified Algebra I* takes a “just in time” approach to review/repair; based upon research with struggling learners, review tasks are strategically placed to help students access prerequisite knowledge and preview upcoming content in the algebra core. Our “repair” strategy is based on research that indicates that learning increases when common mistakes and misconceptions are systematically exposed, challenged, and discussed. In this approach, instructional tasks are designed so that learners confront inconsistencies between their existing beliefs and classroom experiences, then resolve the conflict through targeted practice and reflective discussion. Course developers draw upon the research literature on algebra

teaching and learning to identify common misconceptions and prerequisite knowledge to be addressed.

- **Ongoing, distributed practice.** Struggling students need extra practice opportunities. *Intensified Algebra I* includes several means to promote ongoing practice of skills, concepts, and principles through tasks and problems. Our approach for such practice is based upon long-standing research that supports the use of distributed practice in mathematics. Practice and review in *Intensified Algebra I* are incorporated within the program's curriculum spiral, in homework assignments, and in daily "Staying Sharps" – short problem sets assigned each day that review previously learned material and reinforce prerequisite skills.
- **Social-psychological interventions.** Student motivation, views of intelligence, and engagement are addressed in the *Intensified Algebra I* course. As an example, students' beliefs about their intelligence – as a fixed trait (fixed mindset) or one that can grow over time (malleable mindset) – influence their motivation to engage in academic tasks, and consequently, their success, especially in challenging subjects. Malleable-mindset interventions, which explicitly teach students about the brain, its functions, and that intellectual development is the result of effort and learning, have increased students' achievement in middle school mathematics. Building on these findings, *Intensified Algebra I* incorporates theories of malleable intelligence as well as strategies for goal setting and effective effort; how to learn; and effective communication. These activities have been strategically incorporated into the program.
- **Supports for teachers for enactment of high cognitive demand tasks.** Teachers facilitate students' conceptual development when they engage students in problems and contexts with important mathematics and high cognitive demand tasks and draw explicit attention to connections among ideas, facts, and procedures. A critical element is the extent to which students are actually cognitively engaged with the task at hand and the mathematics they are learning. A key challenge for all teachers, especially those with less experience, is to learn how to enact tasks so that the level of cognitive demand is retained, then orchestrate discussions using students' responses to advance the mathematical learning of the whole class. *Intensified Algebra I* assists teachers with this process by embedding questions and prompts within the student materials that help students get started on activities and focus on important mathematical ideas and relationships. Instructional tasks primarily are designed for partner work (rather than individual or small group work) to promote reflection, discussion and explanations. In addition, the Advice for Instruction (teacher guides) provides detailed information for teachers to support task enactments, e.g., key discussion questions.
- **Tools that help students organize information and support metacognitive awareness.** Metacognitive and cognitive strategies that support students becoming self-directed learners promote students' problem solving capabilities and mathematics achievement. However, struggling learners have difficulty organizing and interpreting

information and complex processes. Thus, a key feature of *Intensified Algebra* is a set of well-defined routines and tools that assists students in making connections within algebra, organizing information, and making their mathematical thinking “visible” to them and to their teachers. For example, the daily routine for introducing homework helps students to summarize learning and make explicit the connections between understandings from the day’s lesson that are needed to successfully complete the homework assignment.

- **Enhanced formative assessments strategies.** The positive effects of formative assessment on mathematics learning are well established. A key aspect of formative assessment involves effectively utilizing evidence gathered about students’ learning to adjust instruction to meet students’ learning needs. Other key aspects involve teachers’ abilities to ask effective questions and engineer high-quality classroom discussions; use mathematics tasks that elicit evidence of learning; and provide feedback that moves learners forward. Typically, individual teachers are responsible for formative assessment. In contrast, *Intensified Algebra I* “offloads” some responsibility for formative assessment onto student and teacher materials. Lessons provide multiple means for students to communicate their thinking, and student and teacher materials contain questions and prompts to help teachers probe students’ understanding and link responses to suggested next steps in instruction. Students complete online assessments, receiving guidance in way of hints and immediate feedback; the Agile Mind reporting tools provide both educators and student with information about task performance. In short, *Intensified Algebra I* contains a comprehensive set of formative assessment activities and tools that provide struggling learners and their teachers with regular and targeted feedback to help them monitor progress, address sources of confusion, and build on the strengths in students’ mathematical knowledge to move them toward more advanced thinking and toward regulation of their own learning.
- **Explicit supports for literacy and language development.** Researchers have begun to understand how literacy and language issues impact learning in mathematics and science classes, particularly with struggling learners, and how to support content learning through the use of simple strategies and tools for language and literacy development. The program promotes better algebra understanding through the use of tools and routines—such as language notes and double- or triple-entry journals, and explicit reading comprehension strategies—to help students build essential academic vocabulary, comprehend and analyze key elements of mathematics problems, explicitly connect different representations of mathematical situations, and reflect upon and communicate their understandings.

Course and lesson structure

Course content is organized into nine instructional units (see the working scope and sequence below), each consisting of roughly four weeks of instruction. Units are subdivided

into three topics – each of which includes a sequence of lessons about specific mathematics content. Typically, the first and third topics in a unit provide 6-7 days each of instruction on specific themes of mathematics, while the middle topic addresses youth development ideas and builds students' algebraic thinking capabilities via engagement in solving non-routine problems.

Each daily lesson is designed for an 80-minute double-block period. A typical lesson has the following components:

- **Daily preview.** Outline of day's activities that explicitly describes what students will be doing and the purpose of each activity. The preview helps students organize their thinking for the lesson and see the connections among lesson components.
- **Opener.** A 5-10 minute daily warm-up routine for transitioning into class work. The goal is to help focus students on the upcoming lesson and access relevant prior knowledge, and to provide teachers with formative assessment data. The warm-ups typically involve mathematics problems involving concepts or skills needed for the lesson but may also involve questions for private reflection and/or partner discussion.
- **Core learning activity.** 25-35 minutes of instruction to promote learning of essential algebra content. Activities feature high-cognitive-demand tasks and are typically situated in real-life context. Online animations of mathematically-important aspects of the tasks are utilized. Tasks are augmented by use of the student activity book to maximize time on task (e.g., so that students do not have to copy problems, tables, etc.) and regular use of routines that provide ways for students to organize and access the content, e.g., routines to support frequent formative assessments (such as use of whiteboards).
- **Process homework.** 10-minute partner routine to review the previous lesson's homework. It promotes communication among students about mathematics and their mathematical thinking, and also teaches students to take ownership of their own learning. Students process their mid-unit and end-of unit assessments with their partners using an expanded version of this routine.
- **Consolidation activity.** 20-25 minutes of instruction designed to review/repair prior knowledge required for upcoming lessons (i.e., preview the algebra content and correct misconceptions), provide additional opportunities for practice to deepen conceptual understanding and skill proficiency, and/or review online and pencil-and-paper assessments. These are typically designed as partner activities.
- **Lesson wrap-up and introduction of homework.** 5-minute routine used to highlight important ideas and activities in the lesson which is accomplished through previewing the lesson's homework assignment.

- **Staying sharp.** A daily set of six short problems that provides distributed practice with algebra and prealgebra skills. Staying Sharps are also used to preview upcoming content; i.e., they help students review relevant prerequisite knowledge and also provide formative assessment data to teachers about their students' knowledge of prerequisite concepts and skills. Staying Sharps are part of the daily homework assignment.
- **Homework.** Roughly 30 minutes of additional work outside of class time to help develop students' confidence and abilities to work independently in mathematics and to provide additional practice with important concepts and skills.

Scope and sequence

The following scope and sequence reflects Version 1 of *Intensified Algebra I*. Subject to change for the 2010-2011 school year.

Unit 1: Getting started

1: Exploring problem-solving strategies

Students begin to experience collaboration as a strategy to solve problems. They share problem-solving strategies as they explore problems that have single and multiple solutions.

2: Getting smarter through problem solving

This topic introduces students to the ideas of malleable intelligence and brain growth through learning. Students continue to develop problem-solving strategies as they extend their understanding of patterns by exploring polygon trains.

3: Problem solving, effort, and your brain

Students learn about the concept of working harder to get smarter, and they apply this idea to learning mathematics. They also learn that effective communication is an interaction between the giver and getter of information. They then apply this understanding as they solve problems in this topic. They continue to explore multiple solution strategies and learn how being able to approach problems in multiple ways can help maintain motivation in problem solving.

Unit 2: Developing mathematical models

4: Representing relationships in multiple ways

This topic develops a key theme of the course: Relationships between variables can be represented using words, tables, graphs, or symbols. Students are formally introduced to different ways to represent patterns and relationships and begin to connect various representations of proportional and non-proportional situations to one another (verbal, numeric, graphical, algebraic). They also extend their understanding of multiple representations in a way that will pay big dividends in Algebra I: They begin to learn to generate other, related representations when given a single representation of a pattern or relationship.

5: Thinking about algebra

Students further develop their problem-solving capabilities and their algebraic thinking by working on a non-routine problem, Consecutive Sums. Students also explicitly explore the use of metacognitive strategies to improve their problem solving and learning.

6: Variables and functions

Students are informally introduced to the concept of function as a dependency relationship between two variables, in which one depends on the other in a systematic way. Students extend their growing

understanding of multiple representations and use them to represent functions involving proportional and nonproportional linear relationships algebraically, numerically, graphically and verbally.

Unit 3: Recognizing linear and nonlinear patterns

7: Linear and non-linear patterns

This topic explores the ideas that linear data show a pattern of constant addition, and, when graphed, the points lie on a line. Students learn to differentiate between the domain and range for a problem and those for a function rule modeling that problem. Then, other patterns that indicate nonlinear relationships between two quantities—specifically, quadratic and exponential relationships—are introduced and represented using tables, graphs, written and verbal descriptions, and algebraic rules.

8: Algebraic thinking and how learning feels

Students further develop their problem-solving capabilities and their algebraic thinking by working on a non-routine problem. Students analyze the feelings they experience as they work on the problem, and learn that confusion is a natural state of learning and that learning often involves backsliding.

9: Linear patterns and proportionality

Students continue to develop problem-solving strategies as they extend their understanding of proportionality by exploring patterns exhibited by proportional relationships. They review the uses of ratios and proportional reasoning in solving real-world problems, use scale factors to enlarge and shrink figures, and express proportional relationships in multiple ways. Real-world applications engage students to explore and make reasonable conjectures while testing their predictions.

Unit 4: Rate of change

10: Rate of change in motion problems

Understanding the rate at which one quantity changes with respect to another is key to understanding how the two quantities are related. In this topic, students explore the concept of rate by analyzing motion over time. Students investigate the rate at which distance changes numerically and graphically.

11. Staying motivated while solving problems

Students also learn that setting goals can support effective effort and learning, even in the face of challenging problems such as the one they tackle in this topic.

12: Exploring rate of change in other situations

This topic deepens student understanding of the central ideas of rate of change. Students discover that they can model data sets that have a constant rate of change with a linear function. Students also learn that not all data are linear, and thus require other models

Unit 5: Linear functions

13: Understanding slope and intercepts

This topic relates the constant rate of change of a linear function, the slope of the line that is the linear function's graph, and the value of m in the linear function rule $y = mx + b$. Students explore this connection using tables, graphs, and function rules. It also develops students' understanding of the x- and y-intercepts of the graph of a linear model and the relationship between the intercepts and the situation being modeled. Students learn to find the values of the intercepts directly from linear function rules expressed in slope intercept form ($y = mx + b$) or standard form ($Ax + By = C$).

14: Building effective communication skills

This topic lays the groundwork for student understanding of the importance of effective communication. Students come to understand effective communication as an interaction between the giver and getter of information. They then apply this understanding as they explore visual representations of data, including scatter plots, and analyze the information contained in a graph.

15: Creating linear models for data

This topic revisits analyzing rate of change to determine whether using a linear model to represent data is appropriate. It also develops the point-slope form for the equation of a line, explicitly connects the point-slope and slope intercept forms, and introduces students to the idea of transformations of functions by transforming the basic function $y=x$ to create linear models for data.

Unit 6: Linear equations and inequalities

16: Solving linear equations

In this topic, students learn how equations are related to functions. The topic explores how different representations of a function lead to techniques to solve linear equations, including tables, graphs, concrete models, algebraic operations, and "undoing" (reasoning backwards).

17: Problem solving in a community of learners

Students develop effective communication skills as they come to understand their importance in their own community of learners.

18: Solving linear inequalities

This topic introduces students to solution techniques for linear inequalities. Students learn to solve with graphs, tables, and algebraic operations.

Unit 7: Systems of linear equations

19: Formulating and solving systems

Systems of linear equations, in which two conditions apply to a situation, and thus must be modeled with two equations, are introduced in this topic. Students learn to set up a system, solve it using graphs and tables, and check the solution for reasonableness.

20: Mindset

Students continue to develop their problem-solving capabilities and algebraic thinking by working on a nonroutine problem, The Icicles Problem. They also explore the idea of "mindset" and how it can affect their success as a student.

21: Other methods for solving systems

Continuing with the exploration of systems of two linear equations, this topic introduces two algebraic methods for solving systems: the substitution method and the linear combination method. Students begin to see when to use each method, and how to interpret the results of each method

Unit 8: Non-Linear functions: Exponential and quadratic functions

22: Exponential functions

This topic builds on students' knowledge of exponential functions by exploring different situations that can be modeled with exponential functions. Students use tables and graphs to contrast the repeated multiplication of exponential patterns with the repeated addition of linear patterns. It also provides a review of laws of exponents and the meanings of negative exponents.

23: Attributions and their effect on problem solving

Students practice identifying attributional styles and reattributing successes and difficulties to controllable factors. They also hone their skills as givers and getters of information as they present their solutions to nonroutine problems.

24: Graphs of quadratic functions

This topic continues the study of transformations on basic functions that began with linear functions. Students build on their previous exposure to quadratic functions as they review the features of the parabola, $y = x^2$, and explore how changes in the values of the constants a and c in $y = ax^2 + c$ affect the graph of the function $y = x^2$.

Unit 9: Quadratic equations

25: Operations on polynomials

This topic explores polynomial operations through a construction scenario. Students learn how to multiply, add, and subtract polynomials using concrete models and analytic techniques. They also learn how to factor trinomials using concrete models and analytic techniques.

26: Solving quadratic equations

This topic focuses on solving quadratic equations that arise from quadratic functions. Students learn to solve these equations by graphing and by factoring and see how the solution methods are connected as they connect the roots of an equation, the x -intercepts of a graph, and the zeros of a function.

27: The quadratic formula

This topic extends the work of the previous topic by introducing students to the quadratic formula as a method for solving quadratic equations. As using this formula sometimes requires students to simplify expressions containing square roots, the connection between the algebra and the geometry of square roots is explored. Students also learn how the value of the discriminant indicates the nature of the solutions.